Small-Space and Streaming Pattern Matching with k Edits

Tomasz Kociumaka Ely Porat Tatiana Starikovskaya

max planck institut informatik

Bar-Ilan University אוניברסיטת בר אילן

ENS | PSL

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Pattern matching

Exact pattern matching

Given two strings, a **pattern** P of length m and a **text** T of length n, find all fragments of T **matching** P.

P `bbaabbb` T `abbaabbbaabbbbbbaabbbbaa`

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P `b b a a b b b` T `a b b a a b b b a a b b b b b a a b b b b a a`

Algorithms: Knuth, Morris, Pratt
1978, SIAM J. Comput. $\mathcal{O}(n+m)$ time

Pattern matching with mismatches

Pattern matching with mismatches

Given a pattern P of length m, a text T of length n, and a **threshold** k, for each position $r \in [m \,..\, n]$, compute the **Hamming distance** $\mathrm{HD}(P, T(r-m \,..\, r])$ **if it does not exceed** k.

$\mathbf{k = 2}$

P `b b a a b b b`

T `a b b a a b b b a a b b b b b b a a b b b b a a`

`– 0 – – – 2 0 2 – – – – – – 2 0 2 – –`

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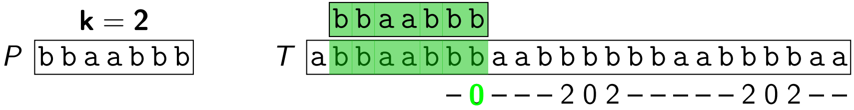

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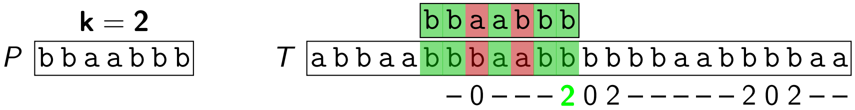

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$$\mathbf{k = 2}$$

P `bbaabbb` T `abbaabbbaabbbbbbaabbbbaa`

`      – 0 – – – 2 0 2 – – – – – – 2 0 2 – –`

⋮   ⋮

Algorithms:

Gawrychowski, Uznański
ICALP 2018

$\widetilde{\mathcal{O}}(n + nk/\sqrt{m})$ time

Pattern matching with edits

Pattern matching with edits

Given a pattern P of length m, a text T of length n, and a threshold k, for each position $r \in [1\,..\,n]$, compute the **edit distance** $\min_{\ell \in [1..r]} \mathrm{ED}(P, T(\ell\,..\,r])$ if it does not exceed k.

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P `b b a a b b b`

T `a b b a a b b b a a b b b b b a a b b b b a a`

– – – – – 2 1 0 1 2 2 1 0 1 2 2 – – 2 1 0 1 2 –

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k = 2

P = b b a a b b b

b b a a b b b

T = a b b a a b b b a a b b b b b b a a b b b b a a

– – – – – 2 1 0 1 2 2 1 0 1 2 2 – – 2 1 0 1 2 –

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P = b b a a b b b

b b a a b b b

T = a b b a a b **b b a a b b b** b b b a a b b b b a a

– – – – – 2 1 0 1 2 2 1 **0** 1 2 2 – – 2 1 0 1 2 –

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T `a b b a a b b b a a b b b b b b a a b b b b a a`

`– – – – – – 2 1 0 1 2 2 1 0 1 2 2 – – 2 1 0 1 2 –`

Algorithms:

Landau, Vishkin
1989, J. Algorithms — $\mathcal{O}(nk)$ time

Cole, Hariharan
2002, SIAM J. Comput. — $\mathcal{O}(n + \frac{nk^4}{m})$ time

Streaming pattern matching

Streaming model:

- Single sequential scan of the text T.

P `bbaabbb` T

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P `bbaabbb` T `abbaabb`

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Streaming pattern matching

Streaming model:

- Single sequential scan of the text T.
- The answer regarding r to be reported while processing $T[r]$.
- Main efficiency measure: size of the working space.

P | b b a a b b b | T | a b b a a b b b a a b b b b b a a b b b b a a |

Streaming pattern matching

Streaming model:

- Single sequential scan of the text T.
- The answer regarding r to be reported while processing $T[r]$.
- Main efficiency measure: size of the working space.
- Deterministic and Las-Vegas algorithms require $\Omega(m \log \sigma)$ bits for exact matching.

P `bbaabbb` T `abbaabbbaabbbbbaabbbbaa`

Streaming pattern matching algorithms

Exact Pattern Matching: $\Omega(\log m)$ bits

Porat, Porat $\mathcal{O}(\log^2 m)$ bits

FOCS 2009

Streaming pattern matching algorithms

Exact Pattern Matching:	$\Omega(\log m)$ bits
Porat, Porat FOCS 2009	$\mathcal{O}(\log^2 m)$ bits
Pattern Matching with Mismatches:	$\Omega(k \log m)$ bits
Porat, Porat FOCS 2009	$\widetilde{\mathcal{O}}(k^3)$ bits
Clifford et al. SODA 2016	$\widetilde{\mathcal{O}}(k^2)$ bits
Golan, Kopelowitz, Porat ICALP 2018	$\widetilde{\mathcal{O}}(k)$ bits
Clifford, K., Porat SODA 2019	$\mathcal{O}(k \log^2 m)$ bits

Streaming pattern matching algorithms

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Pattern Matching with Edits:	$\Omega(k \log m)$ bits
Starikovskaya CPM 2017	$\widetilde{\mathcal{O}}(k^8 \sqrt{m})$ bits
K., Porat, Starikovskaya (this work)	$\widetilde{\mathcal{O}}(k^5)$ bits

Sketches

$X \in \Sigma^{\leq n}$

$\mathrm{ED}(X, Y)$

$Y \in \Sigma^{\leq n}$

Sketches

$X \in \Sigma^{\leq n}$

encoding

$\mathrm{sk}(X)$

$\mathrm{ED}(X, Y)$

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Sketches

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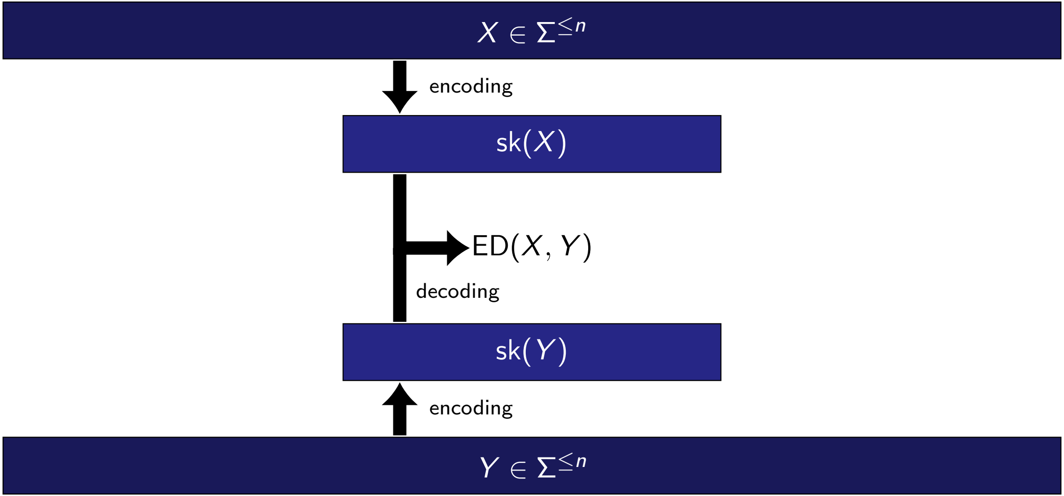
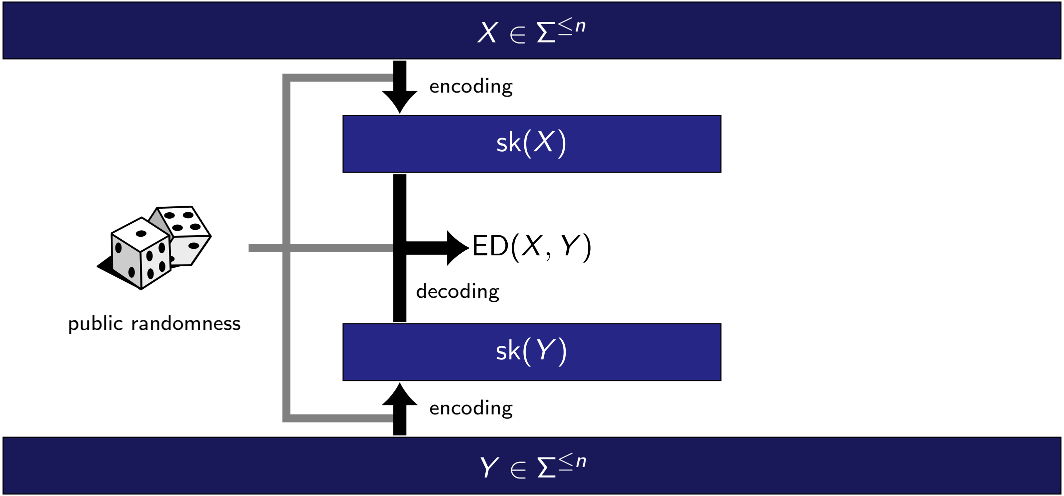
Sketches



$X \in \Sigma^{\leq n}$
encoding
sk(X)
ED(X, Y)
decoding
sk(Y)
encoding
$Y \in \Sigma^{\leq n}$

Sketches



$X \in \Sigma^{\leq n}$
encoding
$\mathsf{sk}(X)$
$\mathsf{ED}(X, Y)$
decoding
public randomness
$\mathsf{sk}(Y)$
encoding
$Y \in \Sigma^{\leq n}$

Known sketches

Efficiently constructible sketches with error probability $n^{-\Theta(1)}$:

Testing equality $X = Y$ (fingerprints):	$\Omega(\log n)$ bits
folklore	$\mathcal{O}(\log n)$ bits

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Lipsky, Porat CPM 2007	$\mathcal{O}(k \log n)$ bits

Known sketches

Efficiently constructible sketches with error probability $n^{-\Theta(1)}$:

Testing equality $X = Y$ (fingerprints):	$\Omega(\log n)$ bits
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Lipsky, Porat CPM 2007	$\mathcal{O}(k \log n)$ bits
Computing ED(X, Y) if at most k:	$\Omega(k \log n)$ bits
Belazzougui, Zhang FOCS 2016	$\widetilde{\mathcal{O}}(k^8)$ bits
Jin, Nelson, Wu STACS 2021	$\widetilde{\mathcal{O}}(k^3)$ bits
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Outline of the talk

Introduction

Streaming exact pattern matching

Streaming pattern matching with edits

Conclusions and open problems

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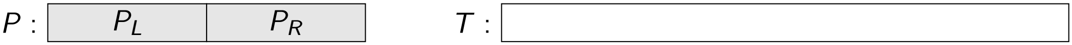
Conclusions and open problems

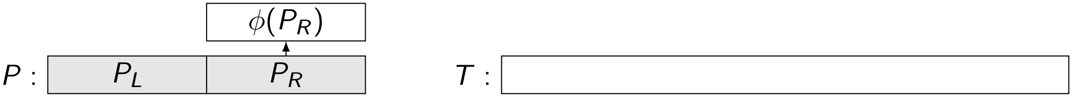
Exact streaming pattern matching: High-level idea

1. Decompose P into two halves P_L and P_R.



Exact streaming pattern matching: High-level idea

1. Decompose P into two halves P_L and P_R.
2. Precompute a **fingerprint** $\phi(P_R)$.



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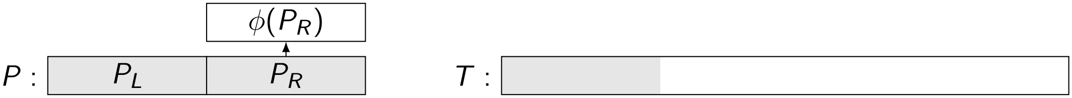

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1. Decompose P into two halves P_L and P_R.
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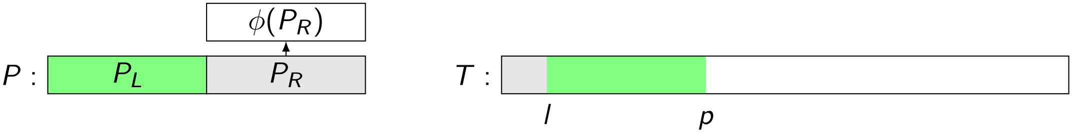
4. Try extending each occurrence $T(l\,..\,p] = P_L$ to an occurrence of P:



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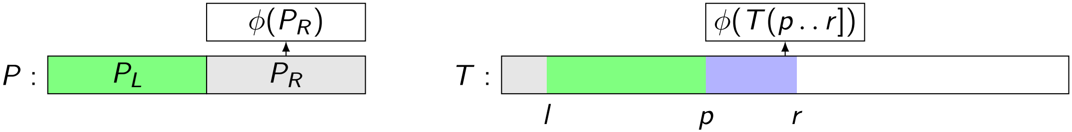

Exact streaming pattern matching: High-level idea

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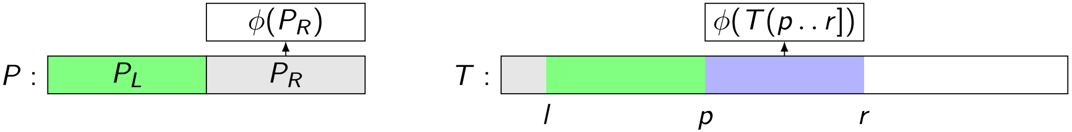
   - Once at $r = p + |P_R|$, compare $\phi(T(p\,..\,r])$ with $\phi(P_R)$.



Exact streaming pattern matching: High-level idea

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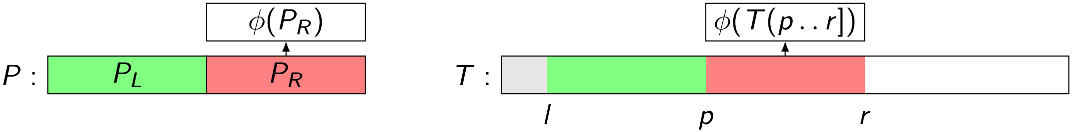
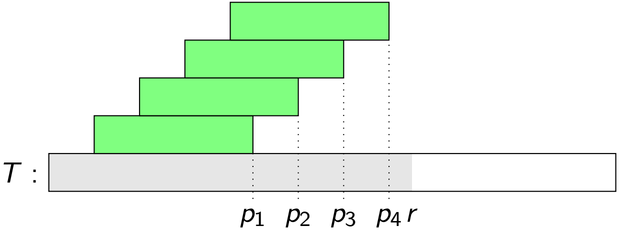


Exact streaming pattern matching: Active occurrences

Active occurrences of P_L

An occurrence $P_L = T(l\,..\,p]$ is **active** if $p \in [r - |P_R|\,..\,r]$.



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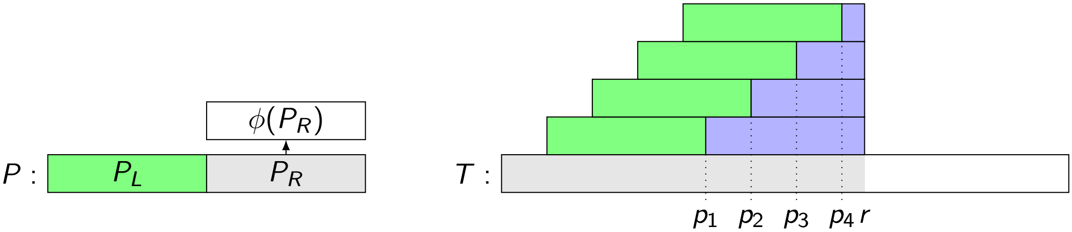

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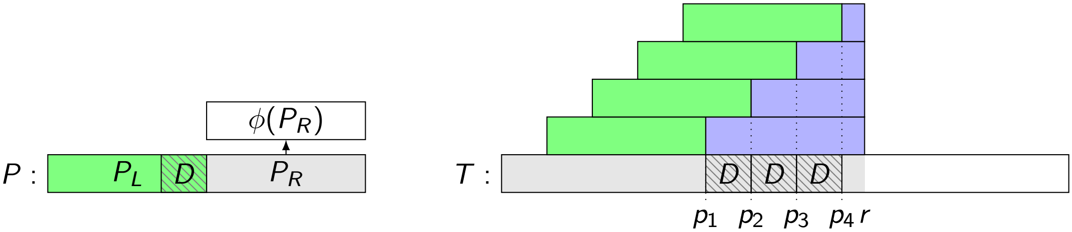

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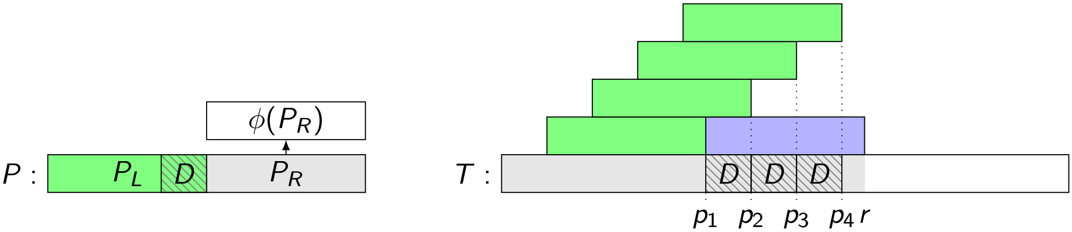

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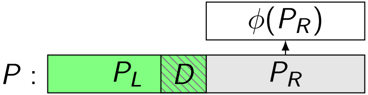

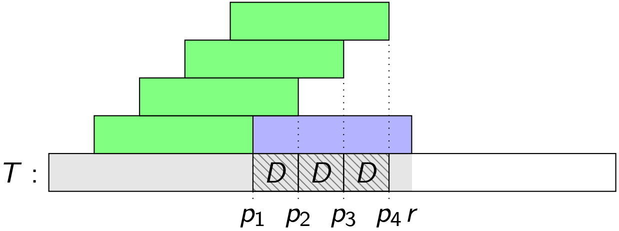

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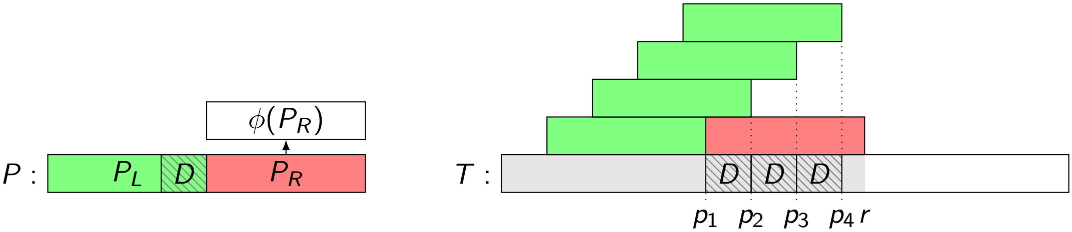

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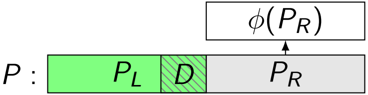

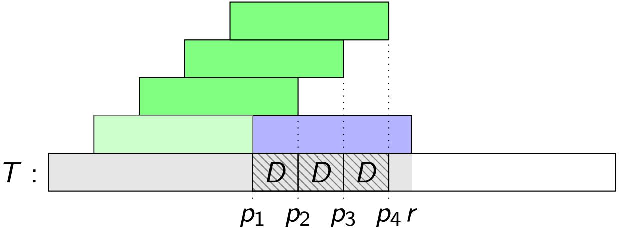

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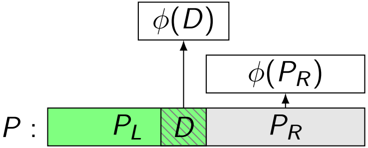

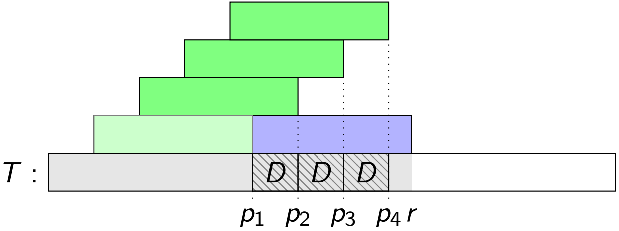

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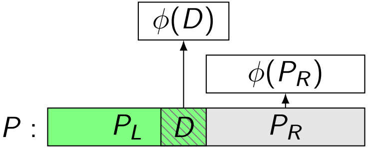
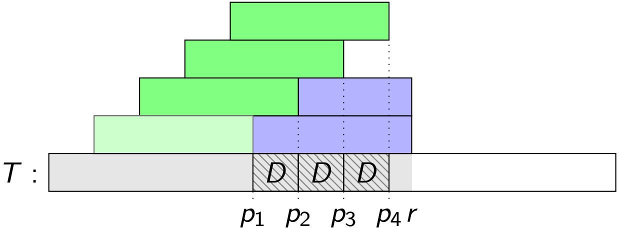
- We maintain $\phi(T(p_1\,..\,r])$ only and, after processing $T(l_1\,..\,p_1]$, use $\phi(D) = \phi(T(p_1\,..\,p_2])$ to derive $\phi(T(p_2\,..\,r])$.



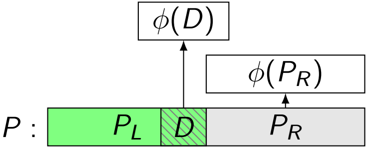


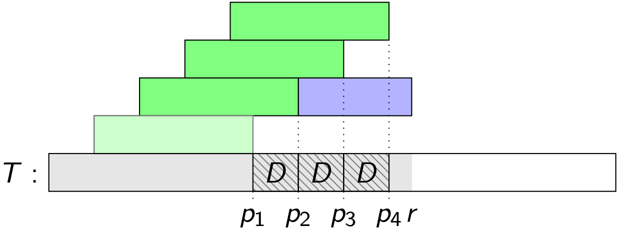
Exact streaming pattern matching: Active occurrences

Active occurrences of P_L

An occurrence $P_L = T(l\,..\,p]$ is **active** if $p \in [r - |P_R|\,..\,r]$.

- For each active occurrence $T(l_i\,..\,p_i]$, we need to maintain $\phi(T(p_i\,..\,r])$.
- The active occurrences form a **chain** with a fixed **difference** $D = T(p_{i-1}\,..\,p_i]$.
- We maintain $\phi(T(p_1\,..\,r])$ only and, after processing $T(l_1\,..\,p_1]$, use $\phi(D) = \phi(T(p_1\,..\,p_2])$ to derive $\phi(T(p_2\,..\,r])$.





Outline of the talk

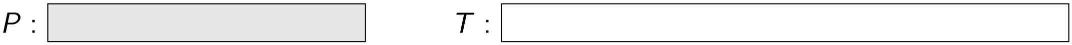
Introduction

Streaming exact pattern matching

Streaming pattern matching with edits

Conclusions and open problems

Streaming pattern matching with edits: High-level idea



Streaming pattern matching with edits: High-level idea

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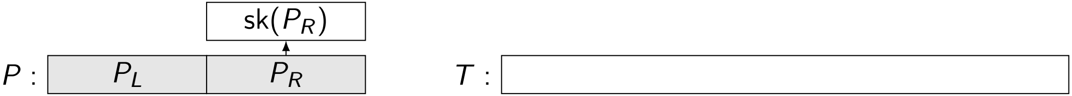

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1. Decompose P into two halves P_L and P_R.
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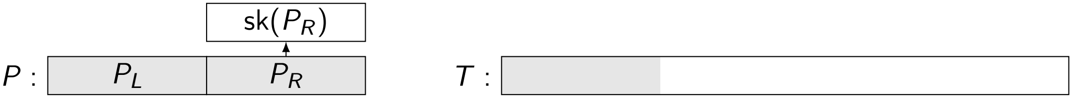

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1. Decompose P into two halves P_L and P_R.
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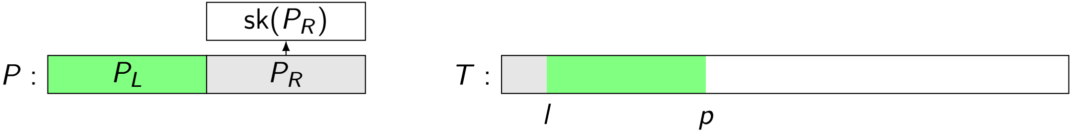
4. Try extending each occurrence $T(l\,..\,p] \approx P_L$ to an occurrence of P:



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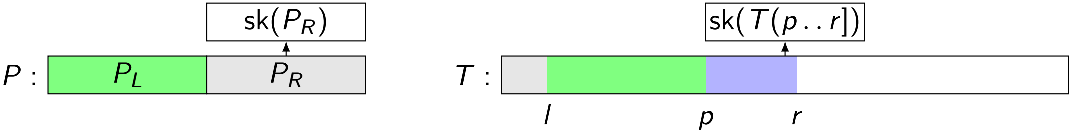

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1. Decompose P into two halves P_L and P_R.
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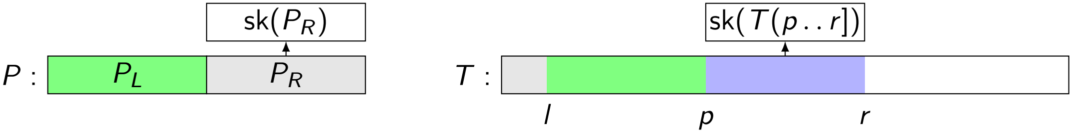
4. Try extending each occurrence $T(l\,..\,p] \approx P_L$ to an occurrence of P:
   - Maintain a **sketch** $\mathrm{sk}(T(p\,..\,r])$.
   - Once at $r \in [p + |P_R| - k\,..\,p + |P_R| + k]$, compare $\mathrm{sk}(T(p\,..\,r])$ with $\mathrm{sk}(P_R)$.



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1. Decompose P into two halves P_L and P_R.
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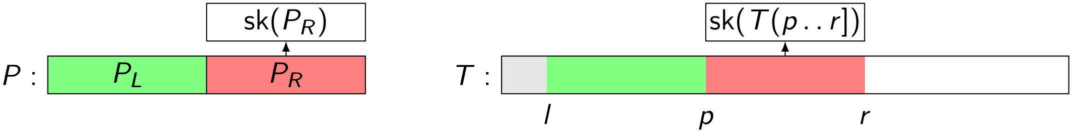
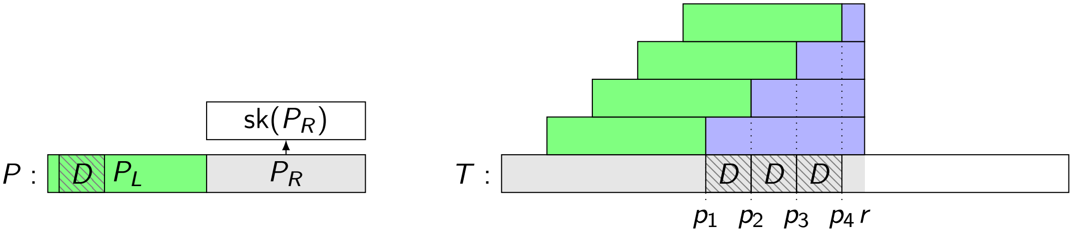


Streaming pattern matching with edits: Active occurrences

Corollary (of Charalampopoulos, K., Wellnitz; FOCS 2020)

Active k-edit occurrences of P_L form $\mathcal{O}(k^3)$ chains whose difference D is among $\mathcal{O}(k)$ prescribed substrings of P_L.

- For each active occurrence $T(l_i \mathinner{.\,.} p_i]$, we need to maintain $\mathrm{sk}(T(p_i \mathinner{.\,.} r])$.



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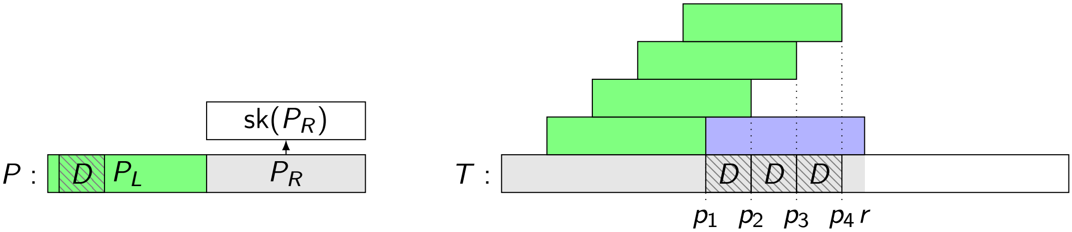

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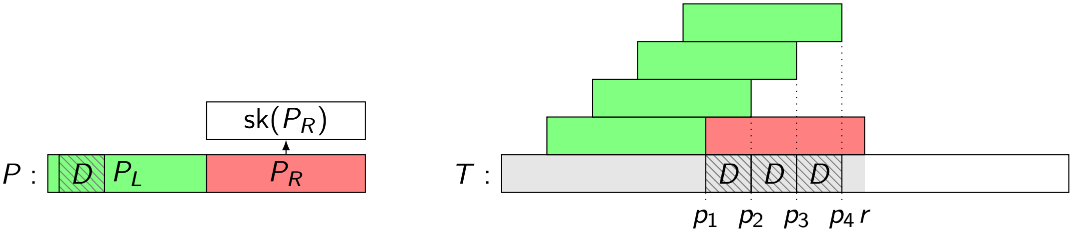

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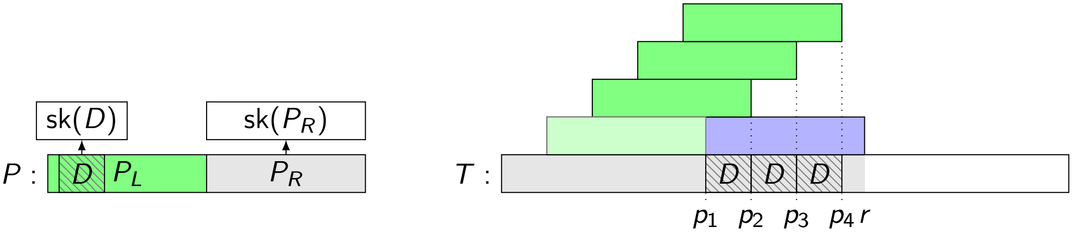

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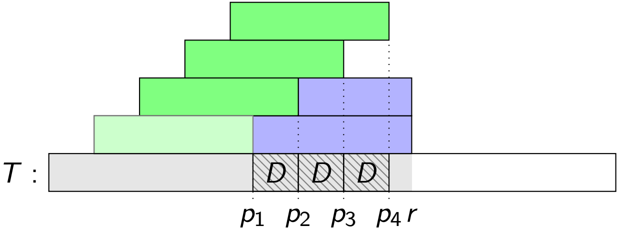

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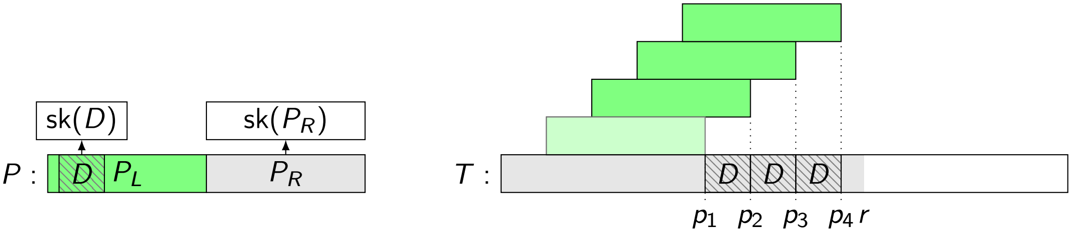

How to circumvent the lack of composable sketches?

1. A careful decomposition $P = P_L P_R$ allows assuming that:
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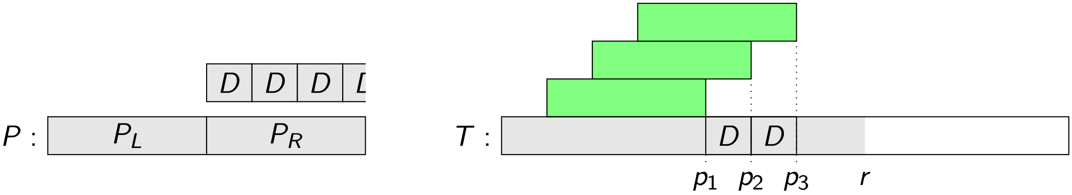

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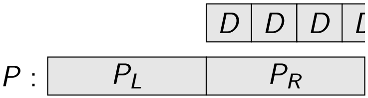

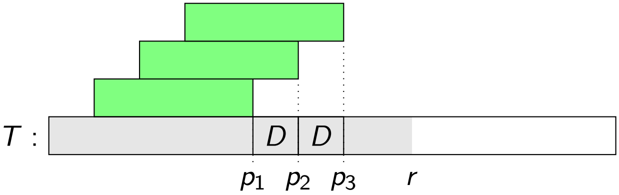

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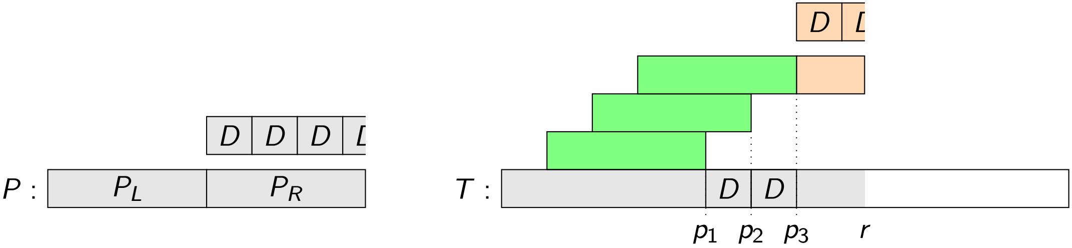

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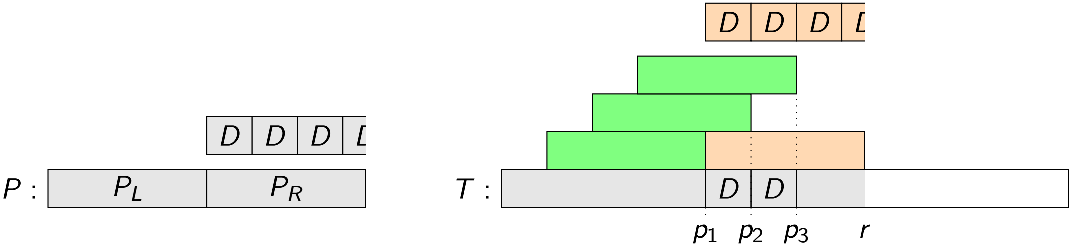

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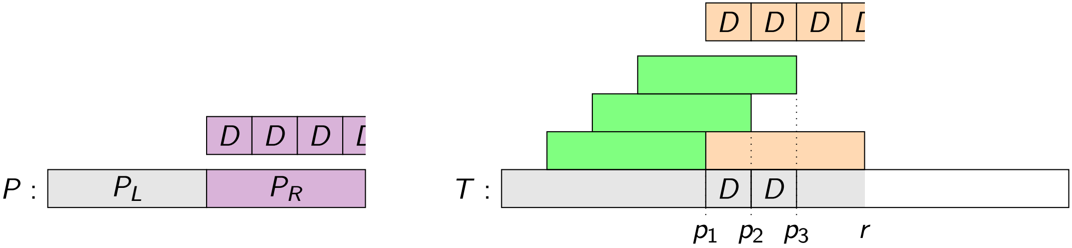

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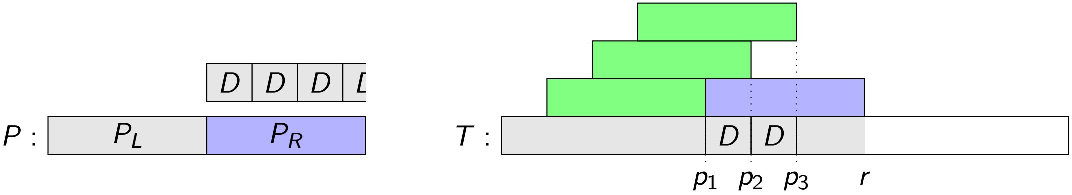

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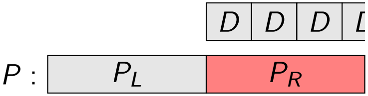

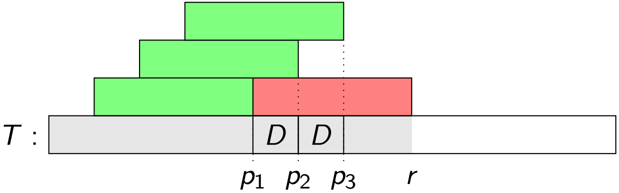

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3. Techniques behind the encoding $\mathcal{E}(X, Y)$:
   - Distinguish **greedy** edit-distance alignments between X, Y;
   - Observe that any two greedy alignments diverge within few **compressible** regions of X, Y.

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Our main result

The pattern matching with edits problem can be solved using an $\widetilde{\mathcal{O}}(k^5)$-space streaming algorithm that costs $\widetilde{\mathcal{O}}(k^8)$ amortized time per character and outputs answers correct w.h.p.

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- Real-time processing of the text.
- Efficient (streaming) preprocessing of the pattern.
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- Lower bounds (so far, we know that $\widetilde{\Omega}(k)$ space is necessary).

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Thank you for your attention!