An implementation of deterministic tree automata minimization

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Abstract

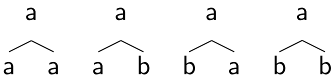
DTAs are highly sparse (most transitions are undefined), equivalence of states depends on multiple inputs, and care must be taken in order to minimize them efficiently.
We fully describe a simple implementation of the standard minimization algorithm that needs a time in $\mathcal{O}(|A|^2)$.

DTA as compact data structure/1

Minimal DTA can store (unranked ordered) tree data efficiently:

1. Each subtree which is common to several trees is assigned a single state.
2. A single state is assigned to groups of subtrees that may appear interchangeably in the collection.

DTA as compact data structure/2

- Sample: 

- States: $\{1, 2, \perp\}$
- Alphabet of labels: $\{a, b\}$
- Accepting states: $\{2\}$
- Transitions $\{(a, 1), (b, 1), (a, 1, 1, 2)\}$.

Bottom-up computations:

$\delta_0(a) = 1$
$\delta_2(a, 1, 1) = 2$

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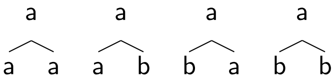
- Sample: 

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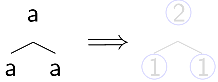
Bottom-up computations: 

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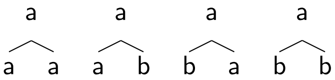

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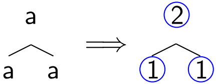

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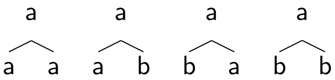
- Sample: 

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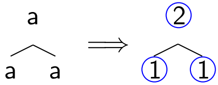
Bottom-up computations: 

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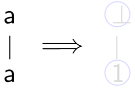

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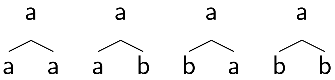
- Sample: 

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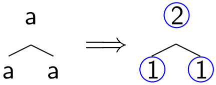
Bottom-up computations: 

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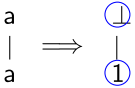

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Congruences in DTA

In a minimal DTA $p \equiv q$ implies

$$p \in F \leftrightarrow q \in F$$

and for all $m > 0$, all $k \leq m$ and all $(\sigma, r_1, ..., r_m) \in \Sigma \times Q^m$

$$\delta_m(\sigma, r_1, \ldots, r_{k-1}, p, r_{k+1} \ldots, r_m) \equiv \delta_m(\sigma, r_1, \ldots, r_{k-1}, q, r_{k+1} \ldots, r_m)$$

DTAs vs DFAs

Compared to DFAs, DTAs

- Lack initial states (transitions with $m = 0$ as $(a, 1)$ and $(b, 1)$ are used as seeds).
- Transitions depend on m states (all siblings).
- Are highly sparse (there are n^m possible inputs of size m, n is num. states).

DFA minimization/1

- DFAs can be minimized in time $\mathcal{O}(kn \log n)$ (k is alphabet size).
- Customary initialization is $\mathcal{O}(|A|^2 \log |A|)$ for sparse DFA.
- A suitable finer initialization leads to $\mathcal{O}(|A| \log |A|)$ cost.

DFA minimization/2

Standard DFA minimization builds the partition $P_0 = \{F, Q - F\}$ and a coarse transition function for all $I, J \in P$:

$$\Delta_{IaJ} = \{(i, a, j) \in \Delta : i \in I \wedge j \in J\}$$

Whenever $s = |\Delta_{Ia}| > 1$, I is split into s classes.

- Finding such (I, a) and updating Δ_{IaJ} is $\mathcal{O}(n)$.
- Number of iterations is $\mathcal{O}(n)$.
- Complexity $\mathcal{O}(kn \log n)$ requires that the largest I subset (that with largest Δ_{IaJ}) remains as I.

Signatures/1

Sparse DFA require:

- Identify useless states and collapse them to $\perp$.
- Initialize the partition P with subsets of states with identical signature and class (accepting or not).

The *signature* of q is

$$\mathrm{sig}(q) = \{a \in \Sigma : \exists (q, a, p) \in \Delta\}$$

Then, only defined transitions are checked.

Signatures/2

In a DTA different definitions of signature are possible

$$
\begin{aligned}
\mathrm{sig}(q) &= \{\sigma \in \Sigma : \exists(\sigma, i_1, \ldots, i_m, j) \in \Delta : \exists k \leq m : i_k = q\} \\
\mathrm{sig}(q) &= \{(\sigma, m) : \exists(\sigma, i_1, \ldots, i_m, j) \in \Delta : \exists k \leq m : i_k = q\} \\
\mathrm{sig}(q) &= \{(\sigma, m, k) : \exists(\sigma, i_1, \ldots, i_m, j) \in \Delta : \exists k \leq m : i_k = q\} \\
\mathrm{sig}(q) &= f(\{(\sigma, i_1, \ldots i_m, j)) \in \Delta : \exists k \leq m : i_k = q\})
\end{aligned}
$$

Homomorphism f is:

$$
f(i_k) = \begin{cases} * & \text{if } i_k = q \\ 0 & \text{if } i_k \neq q \wedge i_k \notin F \\ 1 & \text{otherwise} \end{cases}
$$

Our implementation works will all definitions.

DTA minimization/1

DTA coarse transition function

$$\Delta_{\sigma I_1 \ldots I_m J} = \{(\sigma, i_1, \ldots i_m, j) \in \Delta : i_1 \in I_1, \ldots, i_m \in I_m, j \in J\}$$

If $s = |\Delta_{\sigma I_1 \ldots I_m}| > 1$ at least one I_k needs split. However:

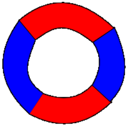
- It is possible that more than one I_k needs split.
- Different $I_{k'} = I_k$ may lead (partially) to same subclasses.
- Which is the largest subset in I_k has nothing to do with the number of transitions in $\Delta_{\sigma I_1 \ldots I_m J}$ (the other I's play).

DTA minimization/2

Useful properties:

- Equivalence is transitive: we define $\mathtt{next}_n(q)$ to return next (or first) element in the equivalence class.
- If two states are not equivalent there exists a pair of distinguishing transitions and at least one leads to $q \neq \perp$.

Graphical interpretation: at least one red-to-blue transition.



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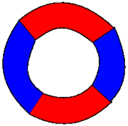


Accessible and coaccessible states/1

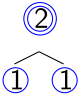
Some definitions:

- State q is *inaccessible* iff $L_A(q) = \emptyset$.
- Accessible state q is *coaccessible* iff there exists $t \in L(A)$ with a subtree s such that $q = A(s)$.
- States which are not coaccessible (and accessible) are *useless*.

For instance, the absorption state $\perp$ is accessible and useless.

Accessible and coaccessible states/2

Accessible states can be found with bottom-up procedure and useless states with a top-down one.
For instance, if $F = \{2\}$ with the computation



- 1 makes 2 accessible,
- 2 makes 1 coaccessible.

Description: algorithm findInaccessible

Input: A DTA $A = (Q, \Sigma, \Delta, F)$
Output: The subset of inaccessible states in A.

1. For all q in Q create an empty list R_q.
2. For all $\tau_n = (\sigma, i_1, ..., i_m, j)$ in Δ do
   - $B_n \leftarrow m$ [Num. of inaccessible pos. in arg(τ_n)].
   - For $k = 1, ..., m$ append n to R_{i_k} [Occurs in $i_1, ... i_m$].
3. $K \leftarrow \{\delta_0(\sigma) : \sigma \in \Sigma\}$; $I \leftarrow Q - K$
4. While $K \neq \emptyset$ and $I \neq \emptyset$ remove a state q from K and for all n in R_q do
   - $B_n \leftarrow B_n - 1$
   - If $B_n = 0$ and $\mathrm{output}(\tau_n) \in I$ then move $\mathrm{output}(\tau_n)$ from I to K. [Whole argument accessible]
5. Return $I - \{\perp\}$.

Description: algorithm findUseless

Input: A reduced DTA $A = (Q, \Sigma, \Delta, F)$ with $F \neq \emptyset$.
Output: The subset of useless states in A.

1. For all q in Q create an empty list L_q.
2. For all $\tau_n = (\sigma, i_1, \ldots, i_m, j)$ in Δ add n to L_j
   [Store n such that j is the output of τ_n (kind of Δ^{-1})].
3. $K \leftarrow F$; $U \leftarrow Q - F$
4. While $K \neq \emptyset$ and $U \neq \emptyset$ remove a state q from K and for all n in L_q and for all i_k in $\{i_1, \ldots, i_m\}$ do
   - If $i_k \in U$ then then move i_k from U to K.
5. Return U.

Description: algorithm minimizeDTA

Input: a DTA $A = (Q, \Sigma, \Delta, F)$ without inaccessible states.
Output: a minimal DTA $A^{\text{min}} = (Q^{\text{min}}, \Sigma, \Delta^{\text{min}}, F^{\text{min}})$.

1. Initialize partition P and queue K.
2. Main loop (refine P).
3. Output A^{min}.

Notation:

- P_n is the partition at iteration n.
- $[q]_n$ is the equivalence class of q in P_n.
- $p \sim_n q \leftrightarrow [p]_n = [q]_n$.

Description: Initialization

- Remove useless states from Q and transitions using them from Δ and set $Q \leftarrow Q \cup \{\bot\}$ and $n \leftarrow 1$.
- For all $(\sigma, i_1, ..., i_m) \in \Delta$ add (σ, m, k) to $\mathrm{sig}(i_k)$ for $k = 1, ..., m$.
- For all $q \in F$ add $(\#, 1, 1)$ to $\mathrm{sig}(q)$. [include acceptance in signature]
- Create an empty set B_{sig} for every different signature sig and for all $q \in Q$ add q to set $B_{\mathrm{sig}(q)}$.
- Set $P_0 \leftarrow (Q)$ and $P_1 \leftarrow \{B_s : B_s \neq \emptyset\}$.
- Enqueue in K the first element from every class in P_1.

Description: Main loop

While K is not empty

1. Remove the first state q in K.
2. For all $(\sigma, i_1, ..., i_m, j) \in \Delta$ such that $j \sim_n q$ and for all $k \leq m$ such that $\delta_m(\sigma, i_1, ..., \mathrm{next}_n(i_k), ..., i_m) \not\sim_n j$
    1. Create P_{n+1} from P_n by splitting $[i_k]_n$ into so many subsets as different classes $[\delta_m(\sigma, i_1, ., i'_k, .., i_m)]_n$ are found for all $i'_k \in [i_k]_n$.
    2. Add to K the first element from every new subset.New splits induced]
    3. Set $n \leftarrow n + 1$.

Description: Output

- Output $(Q^{\min}, \Sigma, \Delta^{\min}, F^{\min})$ with
  - $Q^{\min} = \{[q]_n : q \in Q\}$;
  - $F^{\min} = \{[q]_n : q \in F\}$;
  - $\Delta^{\min} = \{(\sigma, [i_1]_n, \ldots, [i_m]_n, [j]_n) : (\sigma, i_1, \ldots, i_m, j) \in \Delta \wedge [j]_n \neq [\perp]_n\}$.

Analysis/1

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If $p \not\sim_{n+1} q$ there exist $m > 0$, $k \leq m$ and $(\sigma, r_1, ..., r_m, j) \in \Sigma \times Q^{m+1}$ with $r_k = p$ such that

$$\delta_m(\sigma, r_1, \ldots, r_{k-1}, q, r_{k+1} \ldots, r_m) \not\sim_n j.$$

One can assume $j \neq \perp$ (otherwise, one can exchange p and q)

Analysis/2

Define $p^{[1]} = p$ and, for $s > 0$, $p^{[s+1]} = \mathrm{next}(p^{[s]})$. Then, there is $s > 0$ such that

$$\delta_m(\sigma, r_1, \ldots, r_{k-1}, p^{[s]}, r_{k+1} \ldots, r_m) \sim_n j$$

and

$$\delta_m(\sigma, r_1, \ldots, r_{k-1}, p^{[s+1]}, r_{k+1} \ldots, r_m) \not\sim_n j.$$

Analysis/3

The check over all $m > 0$, all $k \leq m$ and all transitions in $\Sigma \times Q^m$ can be limited to those transitions in Δ and every $(\sigma, i_1, ..., i_m, j) \in \Delta$ needs only to be compared with m transitions of the type $(\sigma, i_1, ..., \mathrm{next}(i_k), ...i_m, j')$

Complexity/1

- While K is not empty
  1. Remove the first state q in K.
  2. For all $(\sigma, i_1, \ldots, i_m, j) \in \Delta$ such that $j \sim_n q$ and for all $k \leq m$ such that $\delta_m(\sigma, i_1, \ldots, \mathrm{next}_n(i_k), \ldots, i_m) \not\sim_n j$
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     2. Add to K the first element from every new subset.
     3. Set $n \leftarrow n + 1$.

- A state enters K for every finer class created.
- The refinement process cannot create more than $2|Q| - 1$ different classes (size of a binary tree with $|Q|$ leaves)
- The main loop always removes a state from K; then it performs at most $2|Q| - 1$ iterations.

Complexity/2

- While K is not empty
  1. Remove the first state q in K.
  2. For all $(\sigma, i_1, ..., i_m, j) \in \Delta$ such that $j \sim_n q$ and for all $k \leq m$ such that $\delta_m(\sigma, i_1, ..., \mathrm{next}_n(i_k), ..., i_m) \not\sim_n j$
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     3. Set $n \leftarrow n + 1$.

At every iteration, the internal loop over arguments involves at most $|A|$ iterations.

Complexity/3

- While K is not empty
  1. Remove the first state q in K.
  2. For all $(\sigma, i_1, ..., i_m, j) \in \Delta$ such that $j \sim_n q$ and for all $k \leq m$ such that $\delta_m(\sigma, i_1, ..., \mathrm{next}_n(i_k), ..., i_m) \not\sim_n j$
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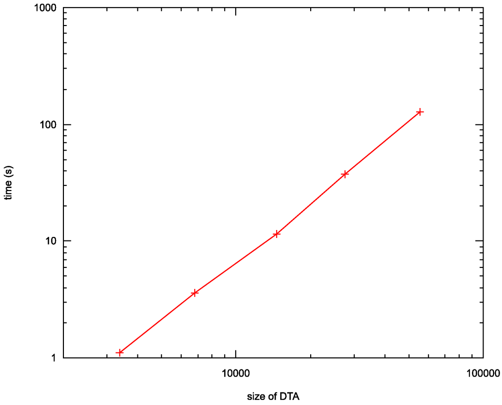
     2. Add to K the first element from every new subset.
     3. Set $n \leftarrow n + 1$.

- If class $[i_k]_n$ is split, its states are classified according to the transition output in less than $|Q|$ steps;
- Updating K adds at most $|Q|$ states.
- Number of splits $< |Q|$; then the conditional block involves at most $|Q|^2$ steps.

Results

Time to minimize acyclic DTA accepting parse trees (up to 2000 trees and 60 labels) from a tree bank.



Results

The time needed to minimize the DTA grows less than quadratically with the size of the automaton (the best fit for this example is $|A|^{1.7}$).

Conclusions and future work

- Simple and efficient minimization of DTA is possible: the search for inconsistent classes can be efficiently performed and undefined transitions and the absorption state can be properly handled.
- A better asymptotic behavior may be still possible.
- We are studyng incremental minimization of DTAs (minimization of a partially minimized automaton).
- Incremental construction (construction of a minimal DTA by adding new trees to the language accepted by an existing one) has also been addressed.