Reducing acyclic cover transducers

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Outline

1. An overview
   - State of the art
   - New algorithms
2. Algorithms
   - Algorithm 1
   - Algorithm 2

Introduction

- Cover automata
  - C. Câmpeanu, N. Sântean, and S. Yu, *Minimal Cover-automata for Finite Languages*, Theoret. Comput. Sci. **267** (2001), 3–16.
  - H. Körner, *A Time and Space Efficient Algorithm for Minimizing Cover Automata for Finite Languages*, Int. J. of Foundations of Comput. Sci. **14** (2003), 1071–1086.
- Extension to cover transducers
  - J.-M. Champarnaud, F. Guingne and G. Hansel, *Cover Transducers for Functions with Finite Domain*, Intern. J. of Foundations of Comp. Sc., 16–5(2005), 851–865.
- Motivation
  - compact representation of dictionaries
  - reducing lexicons in NLP

Preliminaries

- subsequential transducer and subsequential function
- longest common prefix and prefix transducer
- right function, k-function and prefix k-function

Subsequential function and subsequential transducer

- A *subsequential transducer* is a tuple $\mathcal{S} = (\Sigma, \Omega, Q, q\_, F, \mathtt{i}, \mathtt{t}, \cdot, \star)$ where:
  - Σ (resp. Ω) is the *input* (resp. *output*) *alphabet*,
  - Q is the finite set of *states*;
  - $q\_ \in Q$ is the *initial state*,
  - $\mathtt{i} \in \Omega^*$ is the *initialization value*
  - $\mathtt{t} : Q \to \Omega^*$ is the *termination function*,
  - $F = \mathrm{dom}(\mathtt{t})$ is the set of *final states*,
  - $\cdot$ is the *transition function*: $Q \times \Sigma \to q \cdot a \in Q$,
  - $\star$ is the *output function*: $Q \times \Sigma \to q \star a \in \Omega^*$.
- A *subsequential function* $S : \Sigma^* \to \Omega^*$ is realized by a subsequential transducer $\mathcal{S}$

$$S(x) = \mathtt{i}(q\_ \star x)\mathtt{t}(q\_ \cdot x)$$

Example of subsequential transducer

dom(α)	α
a	*abba*
ab	*abbaba*
ba	*babba*
aaa	*abbababba*
abb	*abbababa*
bab	*babbaba*
bba	*bbabba*

Tab. 1. The function α ...

Example of subsequential transducer

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Tab. 1. The function α ...

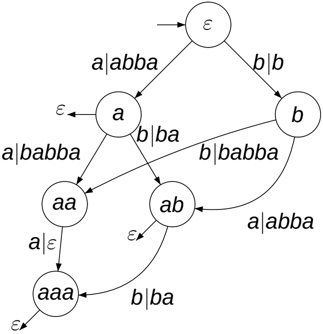
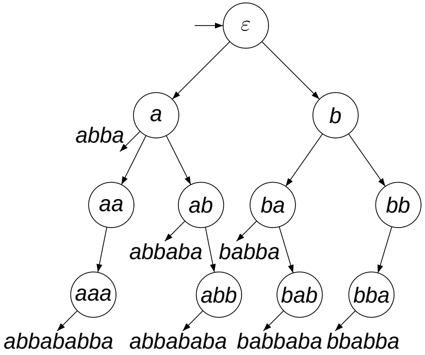


Figure: ... and a subsequential transducer for α.

Longest common prefix and prefix transducer



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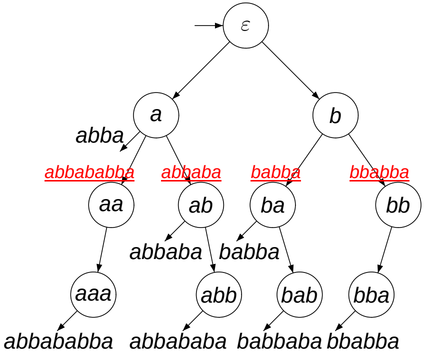

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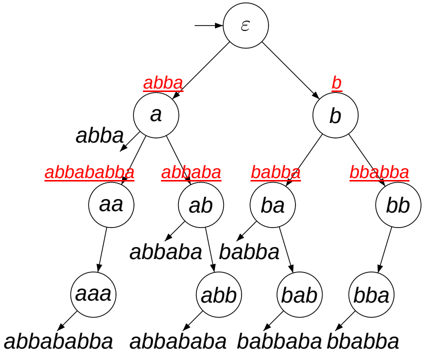

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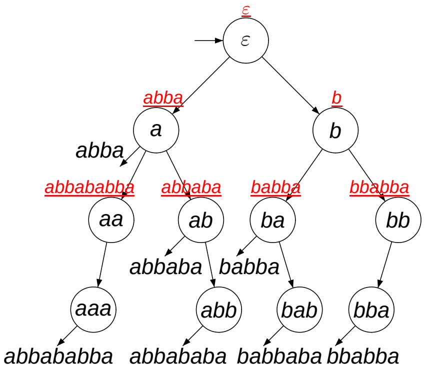

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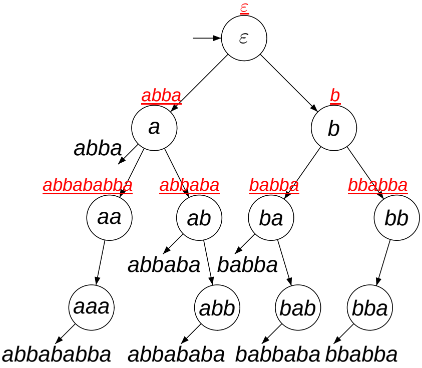


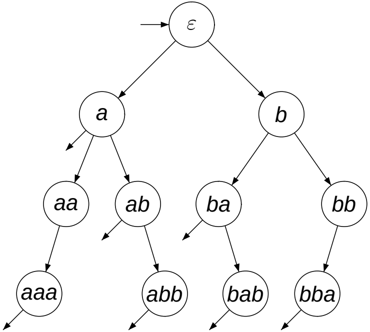
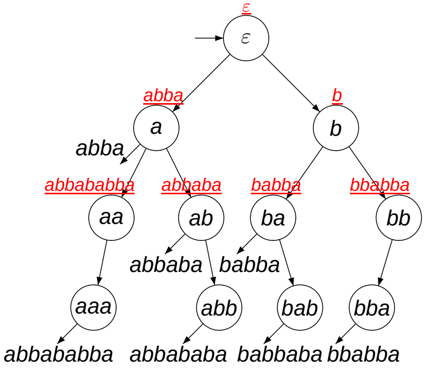
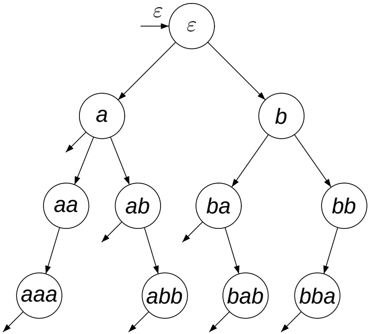


Figure: From the prefix-tree transducer $\mathcal{S}_\alpha$ ...

Longest common prefix and prefix transducer





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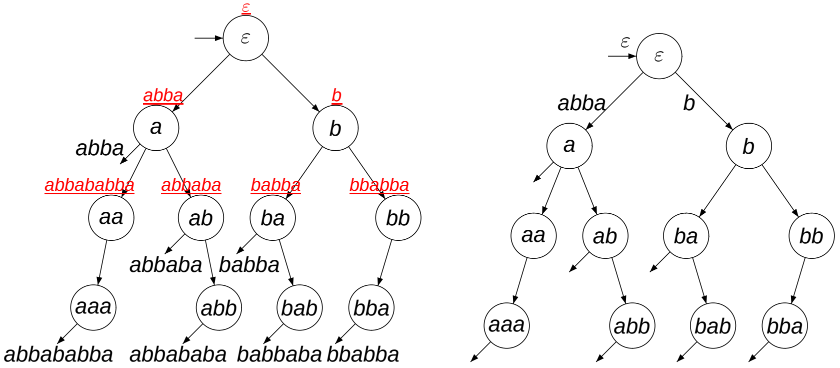

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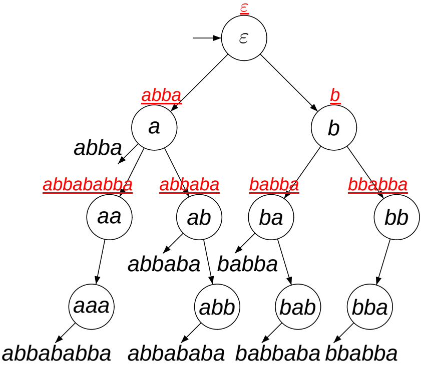

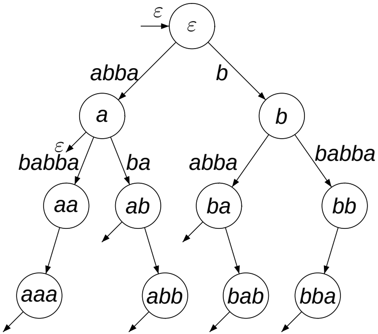

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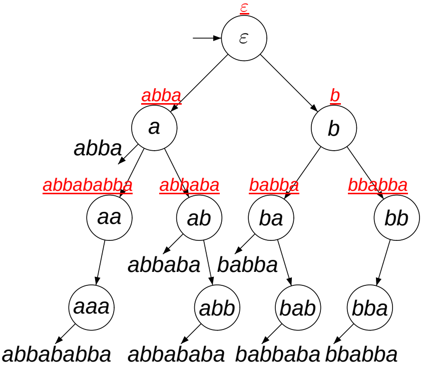

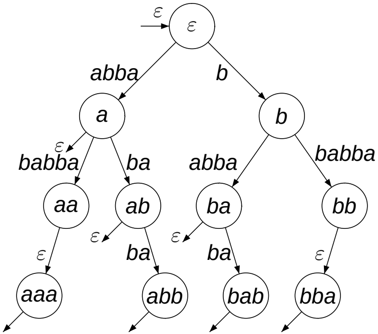

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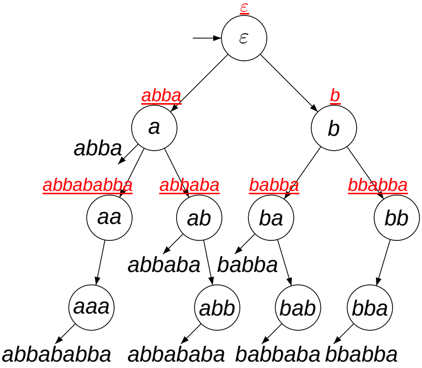

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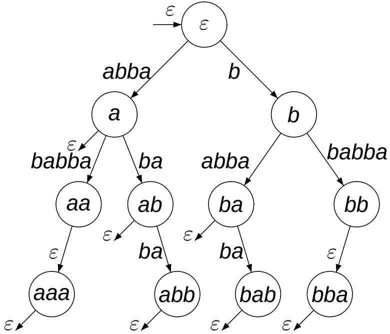

Figure: ... to the prefix transducer $\mathcal{P}_\alpha$.

Right function, k-function and prefix k-function

Right function

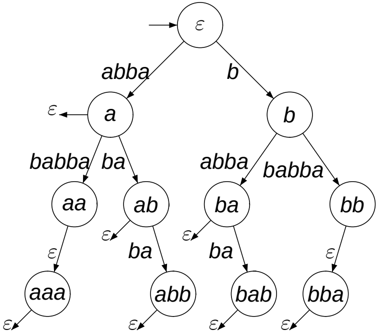

Figure: Prefix-tree transducer.

Right function, k-function and prefix k-function

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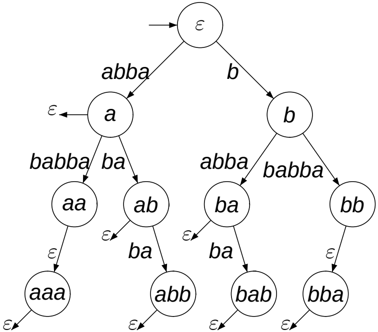

Figure: Prefix-tree transducer.

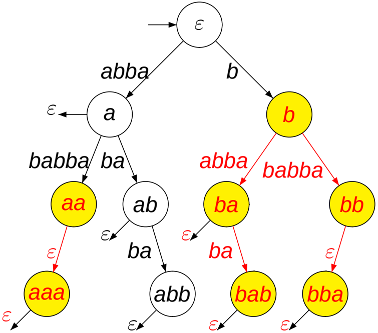

Figure: Right-functions of states *aa* and *b*.

Right function, k-function and prefix k-function

k-function and prefix k-function

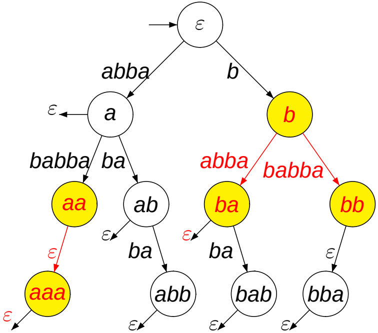

Figure: 1-functions of states *aa* and *b*.

Right function, k-function and prefix k-function

k-function and prefix k-function

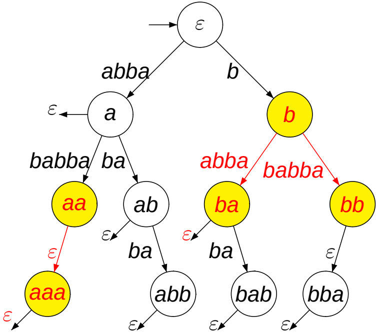

Figure: 1-functions of states *aa* and *b*.

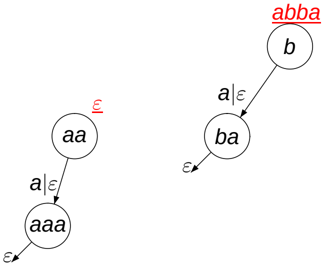

Figure: Prefix 1-functions of states *aa* and *b*.

Overview

- state of the art
  - prefix-tree transducer
  - minimal transducer
  - approximate reduction w.r.t. k-functions
- new algorithms
  - exact reduction w.r.t. k-functions
  - a further reduction via prefix k-functions

Prefix-tree transducer

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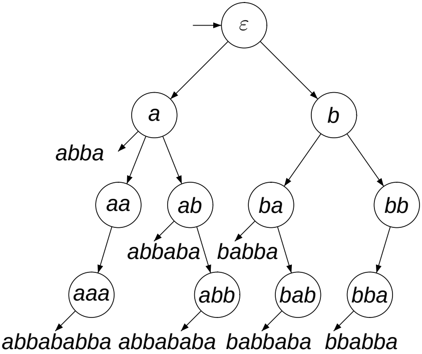


Figure: . . . and its prefix-tree transducer S_α.

Minimal transducer

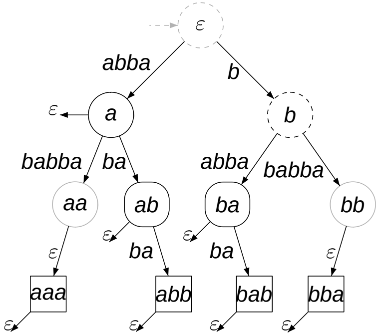


Figure: Partitioning $\mathcal{P}_\alpha$ according to right functions.

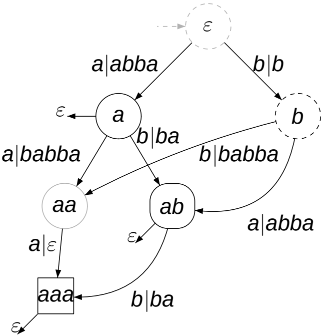


Figure: Minimal transducer $\mathcal{M}_\alpha$.

Approximate reduction w.r.t. k-functions

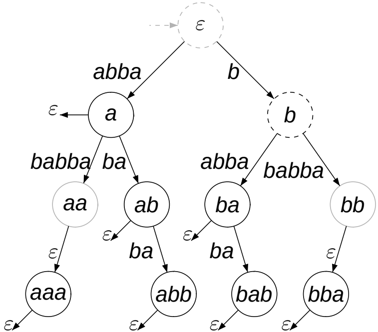
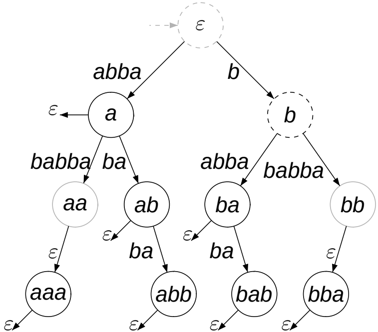


Figure: Approximate k-function partitioning of $\mathcal{P}_\alpha$.

Approximate reduction w.r.t. k-functions



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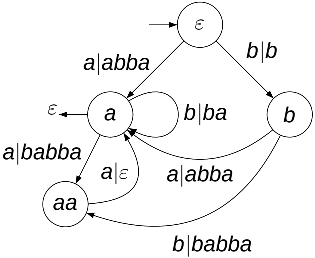


Figure: Reduced cover transducer $\mathcal{R}^1_\alpha$.

Exact reduction w.r.t. k-functions (1)

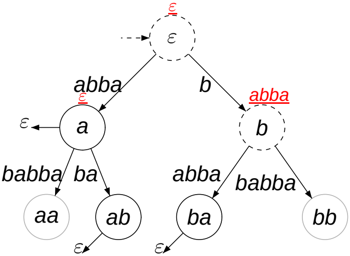
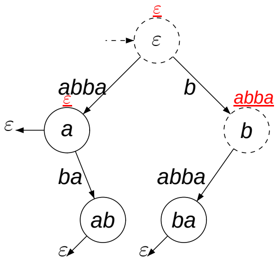


Figure: Construction of the prefix 2-function for state ϵ.

Exact reduction w.r.t. k-functions (1)



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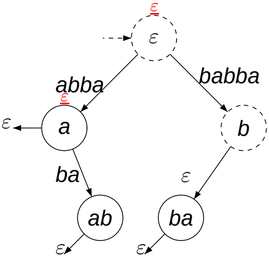

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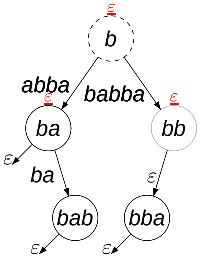


Figure: Prefix 2-function for state b.

Exact reduction w.r.t. k-functions (2)

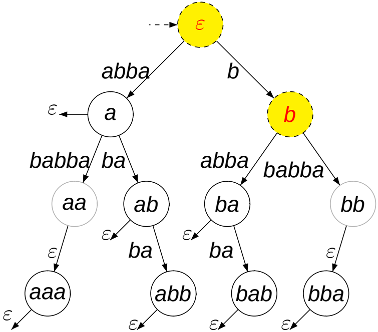


Figure: Exact k-function partitioning of $\mathcal{P}_\alpha$ via prefix k-functions.

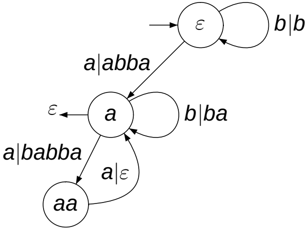


Figure: Reduced cover transducer $\mathcal{R}_\alpha^2$.

A further reduction via prefix k-functions (1)

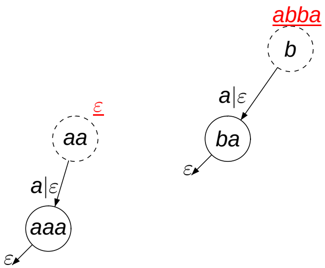
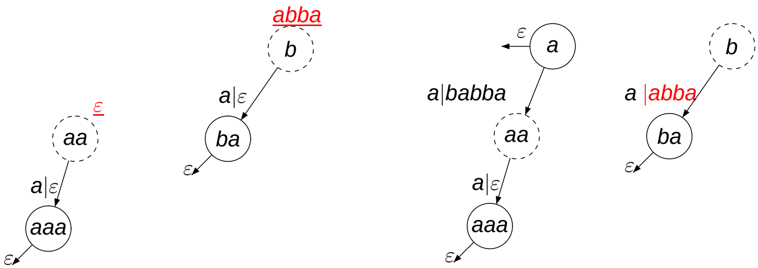


Figure: Prefix 1-functions of states b and aa.

A further reduction via prefix k-functions (1)



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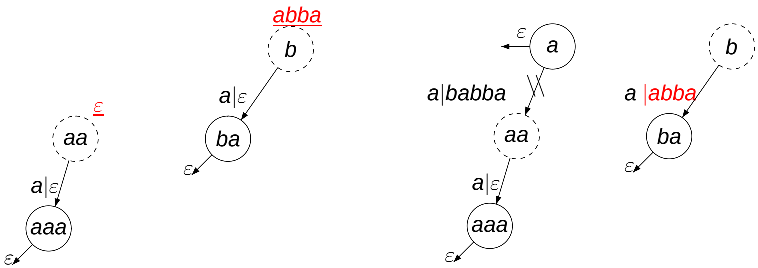

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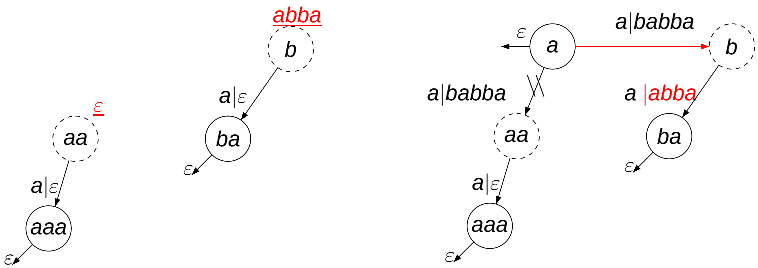

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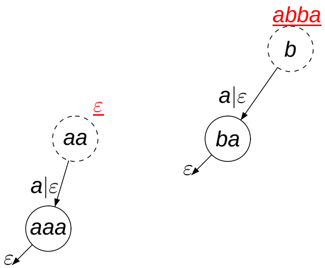

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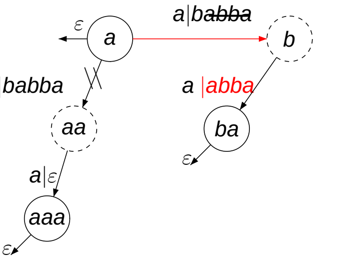


Figure: Merging b and aa in $\mathcal{P}_\alpha$ w.r.t. prefix 1-functions.

A further reduction via prefix k-functions (2)

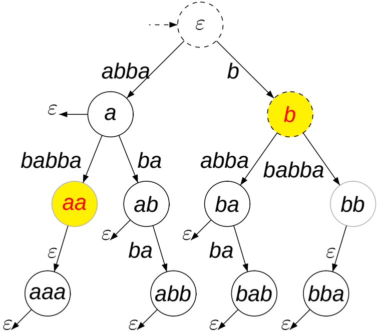
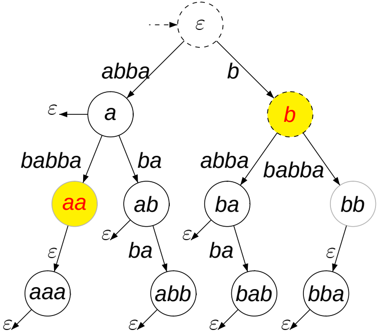


Figure: Exact k-function partitioning of $\mathcal{P}_\alpha$ via prefix k-functions.

A further reduction via prefix *k*-functions (2)



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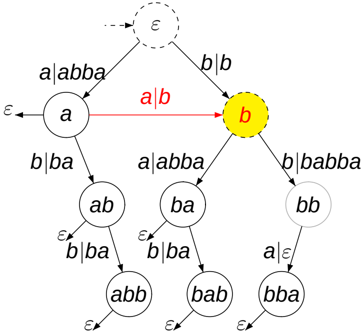

Figure: A further reduction: merging of states *aa* and *b*.

A further reduction via prefix k-functions (3)

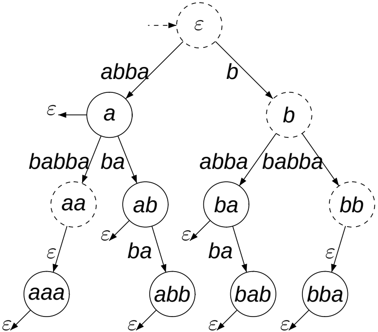

Figure: Partitioning $\mathcal{P}_\alpha$ according to prefix k-functions.

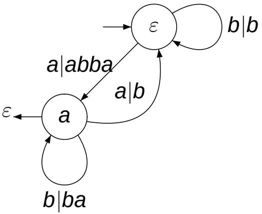

Figure: Reduced cover transducer $\mathcal{R}^3_\alpha$.

Several relations on states

Definitions

Let p, q be two states of Q, and $h = min(height(p), height(q))$

- $p \cong_k q$, $0 \leq k \leq h \iff p$ and q have identical prefix k-functions
- $p \sim_\varepsilon q \iff p$ and q have identical h-functions $\iff$ p and q have identical prefix h-functions and the associated lcps are identical
- $p \sim q \iff p$ and q have identical prefix h-functions and the associated lcp of p is a suffix of the weight of any incoming transition of state q

relations on states

On our Example

	ε	b	aa	bb	a	ab	ba	aaa	abb	bab	bba
$\cong_0$	0	0	0	0	1	1	1	1	1	1	1
$\nu_P(p,0)$	0	0	0	0	ε	ε	ε	ε	ε	ε	ε
$\sim_\varepsilon$	0	0	0	0	1	1	1	1	1	1	1
$\sim$	0	0	0	0	1	1	1	1	1	1	1
$\cong_1$	0	0	0	0	1	1	1				
$\nu_P(p,1)$	$abba$	$abba$	ε	ε	ε	ε	ε				
$\sim_\varepsilon$	2	2	0	0	1	1	1	1	1	1	1
$\sim$	0	0	0	0	1	1	1	1	1	1	1
$\cong_2$	0	0			1						
$\nu_P(p,2)$	ε	ε			ε						
$\sim_\varepsilon$	2	2	0	0	1	1	1	1	1	1	1
$\sim$	0	0	0	0	1	1	1	1	1	1	1

Algorithm 1

1: Input: $\mathcal{P} = (\Sigma, \Omega, Q, q\_, F, \mathtt{i}, \mathtt{t}, \cdot, \star)$, the prefix of the prefix-tree transducer of α.
2: Output: the partition C_k of Q_k w.r.t. $\cong_k$, $\forall 0 \leq k < l$.
3: Comments: $N[p, k]$ is the rank of the class of p in C_k $(0 \leq N[p, k] \leq |C_k| - 1)$.
4: Initializations: $\nu_{\mathcal{P}}(p, 0) = \mathtt{t}[p]$, $\forall p \in Q$; $C_0 = \{Q \setminus F, F\}$.

Algorithm 1

Input: $\mathcal{P} = (\Sigma, \Omega, Q, q\_, F, \mathtt{i}, \mathtt{t}, \cdot, \star)$, the prefix of the prefix-tree transducer of α.
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Initializations: $\nu_{\mathcal{P}}(p, 0) = \mathtt{t}[p]$, $\forall p \in Q$; $C_0 = \{Q \setminus F, F\}$.
for all $k \in 1 \,..\, l - 1$ **do**
  Computation of the relation $\cong_k$
  for all $C \in C_{k-1}$ **do**
    $C^+ = \{p \in C \mid \mathrm{height}(p) \geq k\}$
    for all $p \in C^+$ **do**
      $\nu_{\mathcal{P}}(p, k) = \bigwedge_{a \in \Sigma}(\mathtt{t[p]}, (\mathtt{p} \star \mathtt{a})\nu_{\mathcal{P}}(\mathtt{p} \cdot \mathtt{a}, \mathtt{k} - 1))$
      Computation of the key keya of p
      IF $\nu_{\mathcal{P}}(p, k) = 0$ THEN $A[p] = 0$; $\forall a \in \Sigma$, $B[p, a] = 0$
      ELSE
      $A[p] = \nu_{\mathcal{P}}(p, k)^{-1}\mathtt{t[p]}$
      for all $a \in \Sigma$ **do**
        $B[p, a] = \nu_{\mathcal{P}}(p, k)^{-1}(p \star a)$
      end for
      FI
      $\mathrm{keya}[p] = ((N[p \cdot a, k - 1])_{a \in \Sigma}, A[p], (B[p, a])_{a \in \Sigma})$
    end for
    $\widehat{C^+} = \mathrm{Partition}(C^+, \mathrm{keya})$
    Insert the blocks of $\widehat{C^+}$ into the set C_k.
  end for
end for

Algorithm 2

1: Initializations: $\nu_{\mathcal{P}}(p, 0) = \mathtt{t}[\mathrm{p}]$, $\forall p \in Q$.
2: Initializations: $D_0 = \{Q \setminus F, F_1, F_2\}$, with $F_1 = \{p \in F \mid \nu_{\mathcal{P}}(p, 0) = \varepsilon\}$ and $F_2 = F \setminus F_1$.
3: **for all** $k \in 1 \,..\, l - 1$ **do**
4:     Computation of the partition D_k

Algorithm 2

Initializations: $\nu_{\mathcal{P}}(p, 0) = \mathtt{t}[\mathtt{p}]$, $\forall p \in Q$.
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for all $k \in 1 \mathinner{..} l - 1$ **do**
    Computation of the partition D_k
    for all $D \in D_{k-1}$ **do**
        IF $D^+ = \emptyset$ THEN Insert D into the partition D_k
        ELSE
        for all $p \in D^+$ **do**
            Computation of a partition of D^+ w.r.t. $\cong_k$

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            $\nu_{\mathcal{P}}(p, k) = \bigwedge_{a \in \Sigma}(\mathtt{t[p]}, (\mathtt{p} \star \mathtt{a})\nu_{\mathcal{P}}(\mathtt{p} \cdot \mathtt{a}, \mathtt{k} - 1))$
            $\mathrm{keyb}[p] = \mathrm{Pref}(\nu_{\mathcal{P}}(p, k))$
            Compute $\mathrm{keya}[p]$ according to Lines 11–19 of the Algorithm 1
        end for
        $\widehat{D^+} = \mathrm{Partition}(D^+, \mathrm{keya})$
        $\mathit{linked} = \mathrm{false}$
        for all $C \in \widehat{D^+}$ **do**
            Computation of a partition of C w.r.t. $\sim_\varepsilon$

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      $\mathrm{keyb}[p] = \mathrm{Pref}(\nu_{\mathcal{P}}(p,k))$
      Compute $\mathrm{keya}[p]$ according to Lines 11–19 of the Algorithm 1
    end for
    $\widehat{D^+} = \mathrm{Partition}(D^+, \mathrm{keya})$
    linked $= \mathrm{false}$
    for all $C \in \widehat{D^+}$ **do**
      <u>Computation of a partition of C w.r.t. $\sim_\varepsilon$</u>
      IF $\exists p \in C \mid \mathrm{height}(p) = k$
      THEN $\widehat{C} = \mathrm{Partition}(C, \mathrm{keyb})$
          $\widehat{C} = \{E_1, E_2\}$, with $E_2 = \{p \mid \nu_{\mathcal{P}}(p,k) \succ \varepsilon\}$
          IF $\neg$*linked* THEN $E_1 = E_1 \cup D^-$; *linked* $= \mathrm{true}$ FI
          Insert E_1 into the partition D_k
      ELSE IF $\neg$*linked* THEN $C = C \cup D^-$; *linked* $= \mathrm{true}$ FI
          Insert C into the partition D_k
      FI; **end for**; FI; **end for**; **end for**

Complexity

– $s = sizeof(int)$,
– $t_{max} = \max_{p \in Q}\{\mathrm{length}(\mathtt{t}[p])\}$
– $k_{max} = (|\Sigma| + 1)t_{max} + |\Sigma| + s$

– l is the order of the subsequential function,
– n is the number of states of the transducer,

The Algorithm 2 computes a minimal partition of Q w.r.t. the relation $\sim_\varepsilon$ in $O(k_{max}nl)$ time.

Conclusion

- improvement of a previous algorithm for reducing cover transducers
- further work:
  - develop experimental study
  - find heuristics to compute a partionning as small as possible w.r.t the relation $\sim$
  - extend this study to possibly cyclic cover transducers